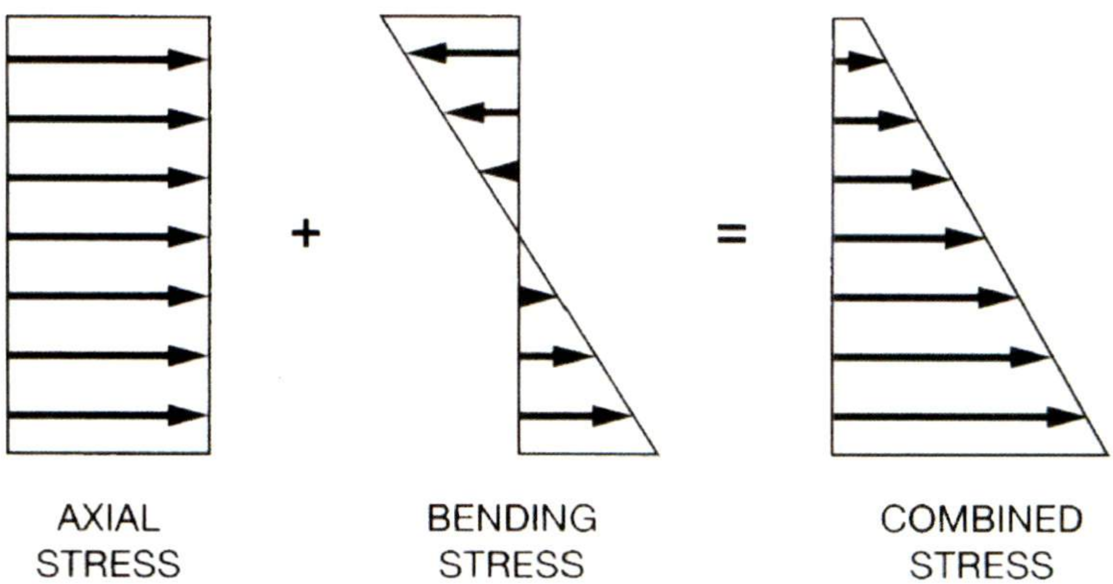
Chapter 18 - Combined Stresses
Reading: Chapter 18 — Pages 653-700

18-1
Introduction

- Formulas for calculating the stresses in members subjected to axial, bending, and shear stresses have been developed in the previous chapters in a separate and individual manner.
- The formulas were derived based on the assumption that the stresses were caused by only one type of loading, and the maximum stress in the member was within the elastic limit of the material, within which stress was proportional to strain.
- In many engineering applications, more than one type of loading may be applied to a member and the member may be subjected to more than one type of stress. Therefore, a technique is needed for finding the combined stress in a member due to several types of loading.
- The method of superposition is used to determine the combined stresses caused by two or more types of loading. Using this method and the fundamental formulas in Table 18-1, the same type of stresses caused by each loading are determined separately. The algebraic sum of these stresses gives the combined stresses caused by all the loadings acting simultaneously. The method of superposition is valid only if the maximum stress is within the elastic limit of the material and if the deformations are small.



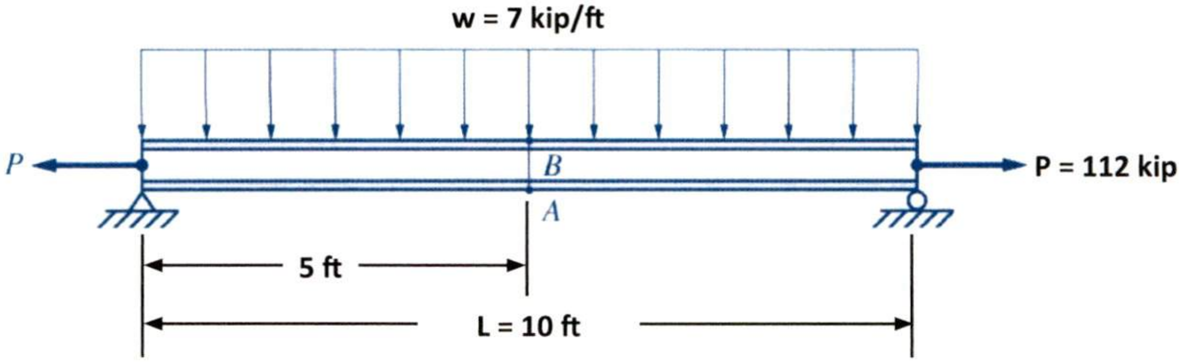
Principle of Super Position with Combined Stress

TABLE 18–1 List of the Fundamental Formulas

Type of Load	*Type of Stress*	*Formula*	*Equation Number*
Axial load	Direct normal stress	$\sigma = \dfrac{P}{A}$	(9–1)
Internal pressure in thin-walled vessels	Circumferential stress	$\sigma_c = \dfrac{Pr_i}{t}$	(9–16)
	Longitudinal stress	$\sigma_l = \dfrac{Pr_i}{2t}$	(9–17)
Beam bending load	Flexural stress	$\sigma = \dfrac{My}{I}$	(14–3)
		$\sigma_{max} = \dfrac{Mc}{I}$	(14–2)
		$\sigma_{max} = \dfrac{M}{S}$	(14–7)
Direct shear load	Direct shear stress	$\tau_{avg} = \dfrac{P}{A}$	(9–4)
Torque in circular shaft	Torsional shear stress	$\tau = \dfrac{T\rho}{J}$	(12–2)
		$\tau_{max} = \dfrac{Tc}{J}$	(12–1)
Beam shear force	Beam shear stress	$\tau = \dfrac{VQ}{It}$	(14–10)
	Maximum shear stress in rectangular section	$\tau_{max} = 1.5\dfrac{V}{A}$	(14–11)
	Maximum shear stress in circular section	$\tau_{max} = \dfrac{4V}{3A}$	(14–12)

Example 18-1 [Converted to U.S. Units]

The wide-flange shape W14 x 68 is used as a simple beam of 10-ft span. The beam is subjected to a uniform load w of 7 kip/ft (including the weight of the beam) and an axial tensile force P of 112 kips. Determine the normal stresses at points A and B, and plot the normal stress variation between A and B.



Solution.

W14x68 Table A-1(a)

$A = 20 \text{ in.}^2$

$I = 723 \text{ in.}^4$

$S = 103 \text{ in.}^3$

Axial Load

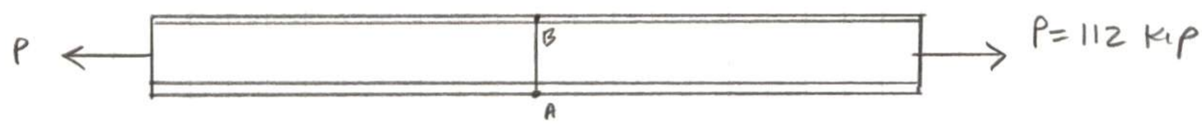
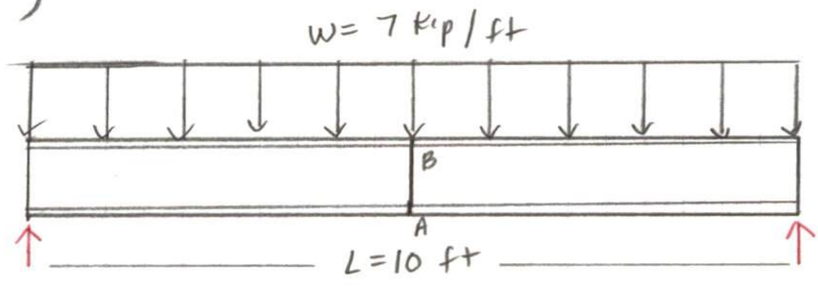


Table 18-1

$$\sigma_A = \sigma_B = \frac{P}{A}$$ (constant tension throughout the beam)

Beam Bending Load



Flexural Stress

$$\sigma_{MAX} = \frac{M}{S}$$

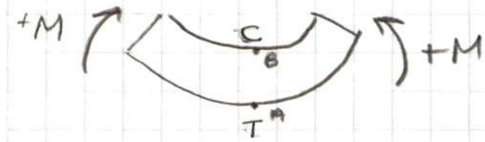
AT midspan

$$M_{MAX} = \frac{wL^2}{8}$$ (Table 13-1, case 4)

For a positive bending moment:

A is in Tension (T)

B is in Compression (C)



$$M_{MAX} = \frac{wL^2}{8} = \frac{7\ kip/ft\ (10ft)^2}{8} = 87.5\ kip \cdot ft \left(\frac{12in}{ft}\right) = 1050\ kip \cdot in$$

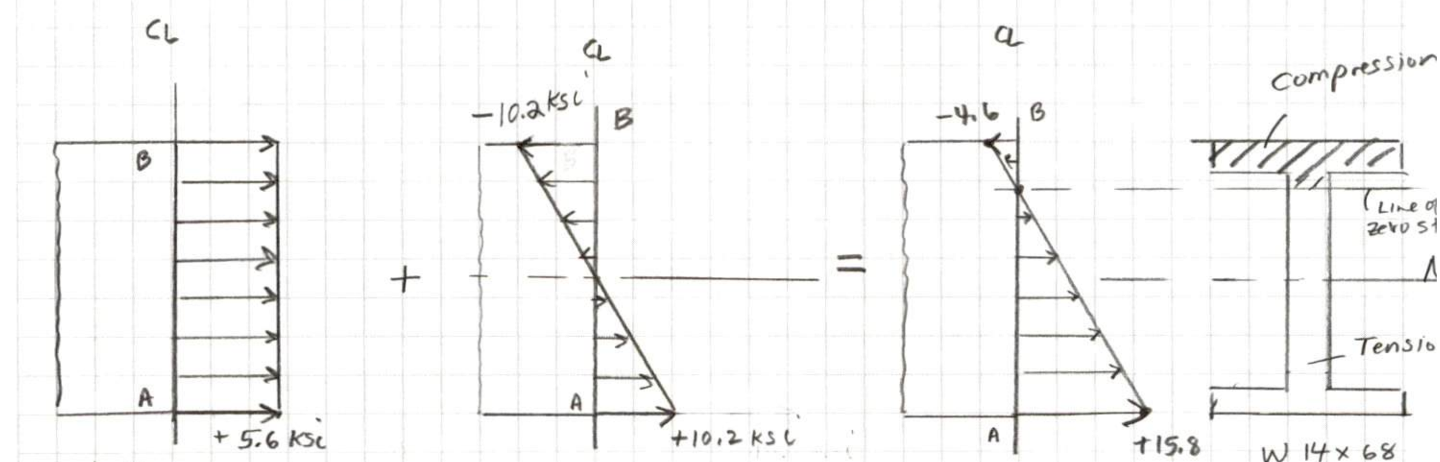
Normal stresses at A

$$\sigma_A = \frac{P}{A} + \frac{M}{S} = \frac{112\ kip}{20\ in^2} + \frac{1050\ kip \cdot in}{103\ in.^3}$$

$$= 5.6\ ksi + 10.2\ ksi = 15.8\ ksi\ (T)$$

Normal stresses at B

$$\sigma_B = \frac{P}{A} - \frac{M}{S} = 5.6\ ksi - 10.2\ ksi = -4.6\ ksi\ (C)$$



Axial Stress

$\frac{P}{A} = +5.6\ ksi$

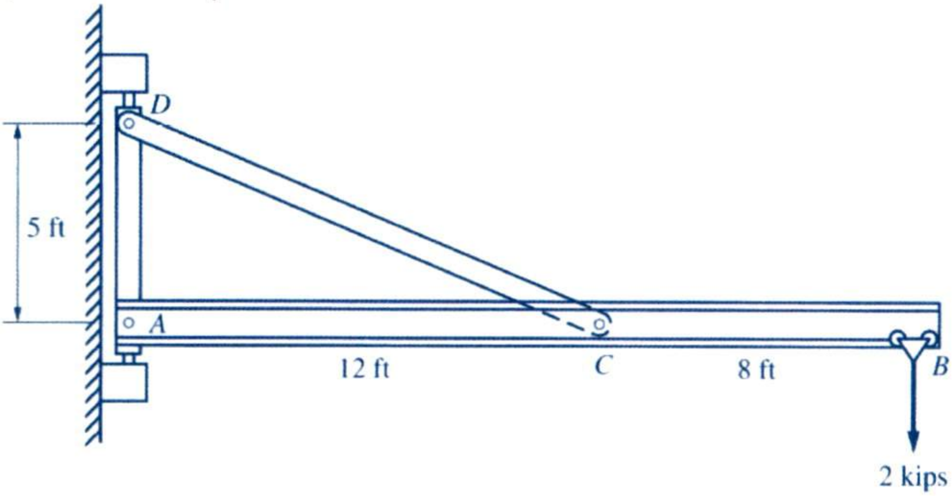
Bending Stress

$\frac{M}{S} = +10.2\ ksi$

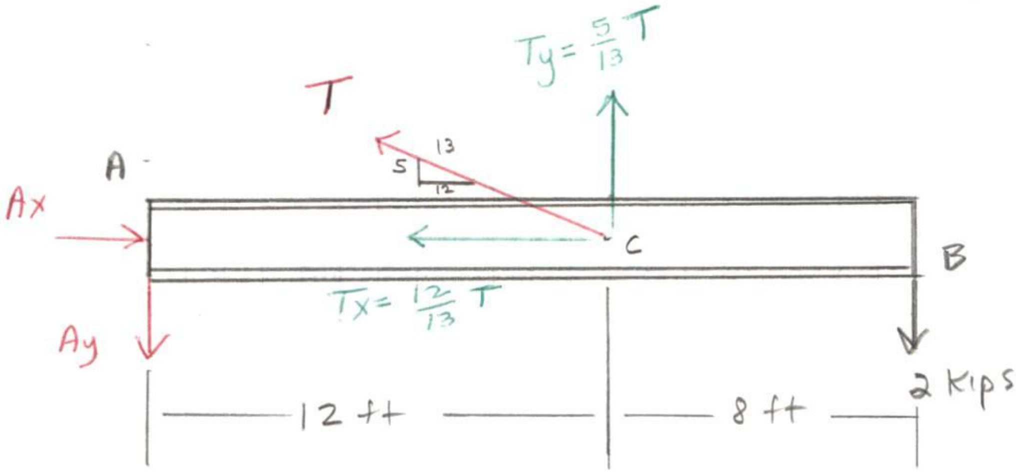
combined stress

Example 18-2

A crane with a swinging arm is designed to hoist a maximum weight of 2 kips. If the allowable compressive stress is 13 ksi, select a W shape for the arm AB.



Solution.



FBD

Equilibrium Equations

$+\circlearrowleft [\Sigma M_A = 0]$ $\quad \frac{5}{13} T\,(12\,ft) - 2\,kips\,(20\,ft) = 0$

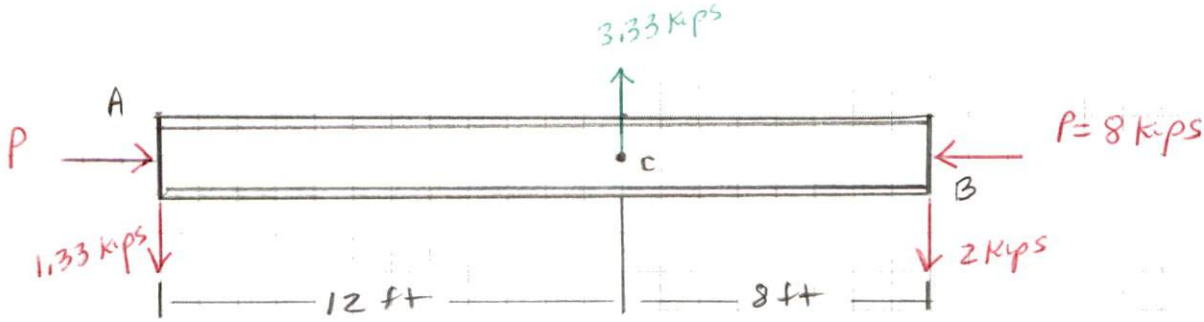
$$T = \frac{13\,(40\,kip \cdot ft)}{5\,(12\,ft)} = 8.67\ kip\ (T)$$

$[\Sigma F_x = 0]$ $\quad A_x - \frac{12}{13} T = 0$

$$A_x = \frac{12}{13}(8.67\ kips) = 8\ kips \rightarrow$$

$[\Sigma F_y = 0]$ $\quad -A_y + \frac{5}{13} T - 2\,kips = 0$

$$A_y = \frac{5}{13}(8.67\ kips) - 2\,kips = 1.33\ kips \downarrow$$



Axial Load

$P = -8$ kips (Constant Compression throughout the Beam)

$$|\sigma| = \frac{P}{A} = \frac{8 \text{ kips}}{A}$$

Beam Bending Load

$$M_{MAX} = -\frac{3.33 \text{ kips}(12 \text{ ft})(8 \text{ ft})}{20 \text{ ft}} = -16 \text{ kip}\cdot\text{ft}\left(\frac{12 \text{ in}}{\text{ft}}\right) = -192 \text{ kip}\cdot\text{in}$$

$$S_{req} = \frac{M}{\sigma_{allow}} = \frac{192 \text{ kip}\cdot\text{in}}{13 \text{ ksi}} = 14.8 \text{ in.}^3$$

Table A-1(a)

Select W 8 x 18 $S = 15.2 \text{ in.}^3$ $A = 5.26 \text{ in.}^2$

$$\left|\sigma_{MAX}^{(c)}\right| = \frac{P}{A} + \frac{M}{S} = \frac{8 \text{ kips}}{5.26 \text{ in}^2} + \frac{192 \text{ kip}\cdot\text{in}}{15.2 \text{ in.}^3}$$

$$= 14.2 \text{ ksi} > \sigma_{allow} = 13 \text{ ksi}$$

W 8 x 18 Too Small

Try, W 8 x 21 $S = 18.2 \text{ in.}^3$ $A = 6.16 \text{ in.}^2$

$$\left|\sigma_{MAX}^{(c)}\right| = \frac{P}{A} + \frac{M}{S} = \frac{8 \text{ kips}}{6.16 \text{ in}^2} + \frac{192 \text{ kip}\cdot\text{in}}{18.2 \text{ in}^3} = 11.8 \text{ ksi} < \sigma_{allow} = 13 \text{ ksi}$$

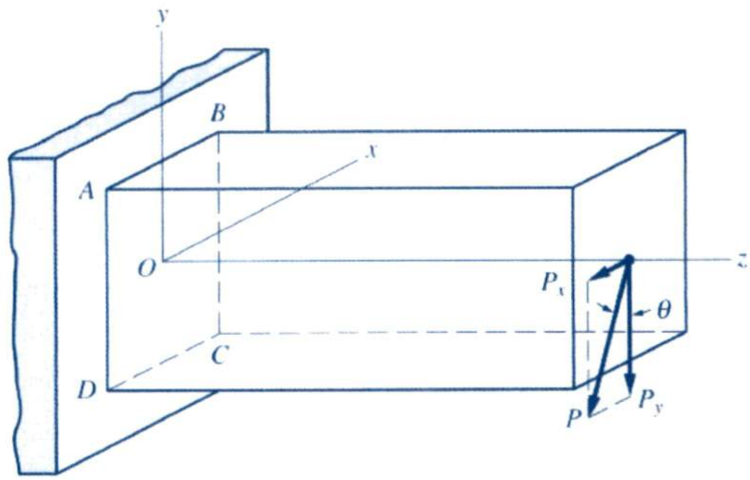
$\boxed{\text{Use, W8x21}}$

18-3
Biaxial Bending

- Problems can arise when a load is inclined at an angle with respect to the vertical plane of symmetry of the beam.
- Load P can be resolved into its horizontal (P_x) and vertical (P_y) components, in the directions of the two axes of symmetry at the section on the free end.

> P_y causes bending about the horizontal axis
> P_x causes bending about the vertical axis

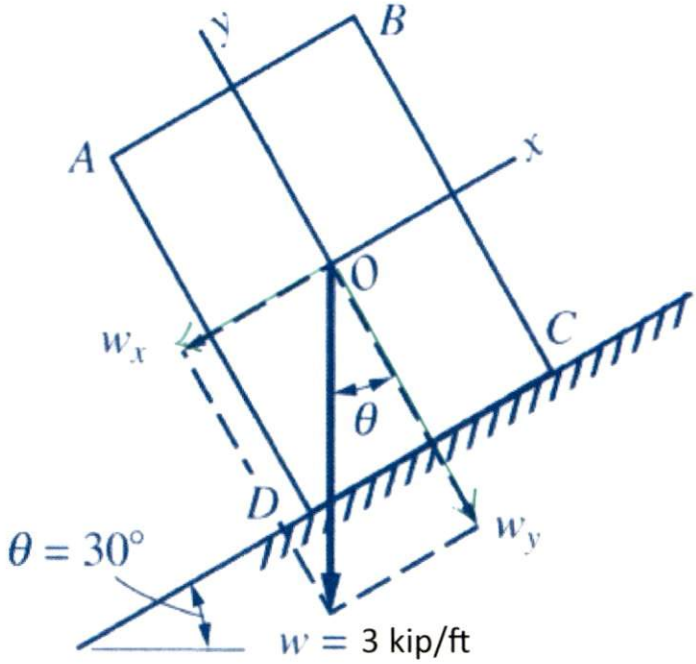
$\left.\begin{array}{l} \\ \\ \end{array}\right\}$ Biaxial Bending



- Either type of bending causes normal stresses along the longitudinal direction and superposition can be applied.
- The bending about each axis is calculated separately and the results are added algebraically.

Example 18-3 [Converted to U.S. Units]

A simply supported timber beam of 10-ft span has a rectangular cross-section with a nominal size of 6-in x 8-in. The beam carries a uniform load w of 3 kip/ft and is supported at the ends in the tilted position shown in Fig. E18-3. Determine the maximum flexural stresses in the beam.



Solution.

$$w_x = w \sin\theta = (3\ \text{kip/ft})(\sin 30°) = 1.5\ \text{kip/ft}$$

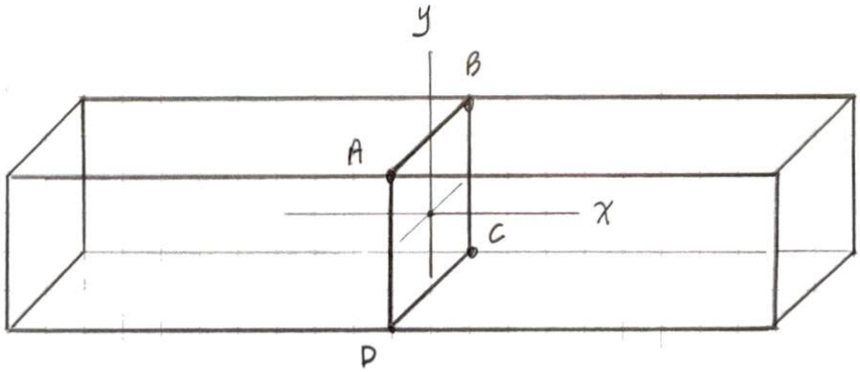
$$w_y = w \cos\theta = (3\ \text{kip/ft})(\cos 30°) = 2.6\ \text{kip/ft}$$

From Table 13-1, case 4 (uniform Load)

$$M_{MAX} = \frac{wL^2}{8} \quad \text{(at the midspan)}$$

$$M_x = \frac{w_y L^2}{8} = \frac{2.6\ \text{kip/ft}\ (10\ \text{ft})^2}{8} = 32.5\ \text{kip}\cdot\text{ft}$$

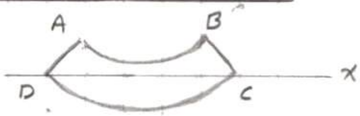
$$M_y = \frac{w_x L^2}{8} = \frac{1.5\ \text{kip/ft}\ (10\ \text{ft})^2}{8} = 18.75\ \text{kip}\cdot\text{ft}$$



Nominal
6 in × 8 in
Rectangular Beam

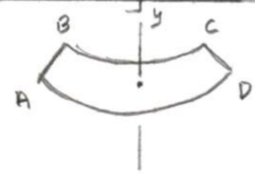
Bending about the x-axis

CD (T)
AB (C)



Bending about the y-axis

AD (T)
BC (C)



From Table A-6(a)

$$S_x = 51.6 \text{ in}^3$$

$$\sigma_1 = \frac{M_x}{S_x} = \frac{32.5 \text{ kip}\cdot\text{ft}\left(\frac{12 \text{ in}}{\text{ft}}\right)}{51.6 \text{ in}^3}$$

$$= 7.6 \text{ ksi}$$

For a Rectangular Section

$$S_y = \frac{bh^2}{6} = \frac{7.5 \text{ in}\,(5.5 \text{ in})^2}{6} = 37.8 \text{ in.}^3$$

$$\sigma_2 = \frac{M_y}{S_y} = \frac{18.75 \text{ kip-ft}\left(\frac{12 \text{ in}}{\text{ft}}\right)}{37.8 \text{ in.}^3}$$

$$\sigma_2 = 6 \text{ ksi}$$

Combined Stresses

$$\sigma_A = -\sigma_1 + \sigma_2 = -7.6 \text{ ksi} + 6 \text{ ksi} = -1.6 \text{ ksi (C)}$$

$$\sigma_B = -\sigma_1 - \sigma_2 = -7.6 \text{ ksi} - 6 \text{ ksi} = -13.6 \text{ ksi (C)}$$

$$\sigma_C = \sigma_1 - \sigma_2 = 7.6 \text{ ksi} - 6 \text{ ksi} = 1.6 \text{ ksi (T)}$$

$$\sigma_D = \sigma_1 + \sigma_2 = 7.6 \text{ ksi} + 6 \text{ ksi} = 13.6 \text{ ksi (T)}$$

Maximum Flexural Stresses

$$\sigma_{MAX}^{(T)} = 13.6 \text{ ksi}$$

$$\sigma_{MAX}^{(C)} = -13.6 \text{ ksi}$$